[Time:3:00 Hrs.] **[Marks: 80]**

Please check whether you have got the right question paper.

N.B:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Scientific calculator can be used.

Q.1 a) i) Define Uniformly Bounded Set **10**

ii) Define Equicontinuous Set of functions

iii) Prove " If on a bounded Interval I, $F = \{f\}$ is an infinite, uniformly bounded, equicontinuous set of functions. Then F contains a sequence $\{f_n\}$, $n = 1,2,3,\ldots$ which is uniformly convergent on I."

b) Attempt **any Two** of the following: **10**

i) Show that for the Second order ODE $y'' + p(x)y' + q(x)y = 0$,if $y = x$ is a solution of the ODE then $p(x) + xq(x) = 0$. **5**

ii) Solve the system of equation **5**

$$\frac{dx}{dt} = -x + y$$
$$\frac{dy}{dt} = -y$$

for $x(0) = y(0) = 1$

iii) Let $W(t)$ denote the Wronskian of solutions to the Linear Homogeneous System of Equations **5**

$$\frac{dx}{dt} = a_1\, x + b_1\, y$$
$$\frac{dy}{dt} = a_2\, x + b_2\, y$$

Prove that $W(t)$ is identically zero or never zero on the Interval I.

Q.2 a) Let $L(y) = y'' + a_1 y' + a_2 y = 0$ where a_1, a_2 are constants. Prove the following: **10**

i) If r_1, r_2 are distinct roots of the auxiliary equation then the functions $\phi_1(x) = e^{r_1 x}, \phi_2(x) = e^{r_2 x}$ are solutions of $L(y) = 0$.

ii) If r is a repeated root of the auxilary equation then the functions $\phi_1(x) = e^{rx}, \phi_2(x) = xe^{rx}$ are solutions of $L(y) = 0$.

	b)	Attempt **any Two** of the following:	**10**
	i)	If $y = 3e^{2x} + e^{-2x} - \alpha x$ is the solution to the Initial Value Problem $(D^2 + \beta)y = 4\alpha x, y(0) = 4, y'(0) = 1$ where $\alpha, \beta \in \mathbb{R}$. Find α, β.	**5**
	ii)	Check whether the functions $\phi_1(x) = 1, \phi_2(x) = x, \phi_3(x) = x^3$ are linearly Independent or Dependent on $-\infty < x < \infty$.Justify your answer.	**5**
	iii)	If $y(x)$ is a solution of the differential equation $\frac{dy}{dx} + \frac{2x+1}{x} y = e^{-2x}$ then calculate the value of $\lim_{x\to\infty} y(x)$.	**5**
Q.3	**a)**	Let $\phi_1, \phi_2, , \phi_n$ be n solutions of $L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \cdots .. + a_n(x)y = 0$ on an interval I and let x_0 be any point in I then prove that $$W(\phi_1, \phi_2, \ldots, \phi_n)(x) = \exp[-\int_{x_0}^{x} a_1(t)dt]\, W(\phi_1, \phi_2, \ldots, \phi_n)(x_0)$$	**10**
	b)	Attempt **any Two** of the following:	**10**
	i)	Find the solution of the differential equation $\frac{d}{dx}\left(x\frac{dy}{dx}\right) = x, y(1) = 0, \frac{dy}{dx}(x = 1) = 0$.	**5**
	ii)	Derive the conditions for which e^{mx} is a solution of the Second Order Ordinary Differential Equation $y'' + P(x)y' + Q(x)y = 0$. Derive conditions for $m = 1,2$.	**5**
	iii)	Solve the Cauchy Euler Equation $x^2y''(x) - 2y(x) = 0$ satisfying $y(1) = 1, y(2) = 1$.	**5**
Q.4	**a)**	Consider the Differential equation $\frac{d}{dx}[p(x)\frac{dy}{dx}] + [q(x) + \lambda r(x)]y = 0$ where p, q, r are continuous functions of x with p having continuous first order derivative, $p(x) > 0, r(x) > 0\ \forall x$ and λ is a constant. Also $a_1 y(a) + a_2 y'(a) = 0$. $b_1 y(b) + b_2 y'(b) = 0$ where a_1, a_2, b_1, b_2 are real constants with $a_1^2 + a_2^2 \neq 0$ and $b_1^2 + b_2^2 \neq 0$ then show that for distinct Eigenvalues λ_m and λ_n, the corresponding Eigen functions ϕ_m and ϕ_n are orthogonal with respect to the weight function $r(x)$.	**10**
	b)	Attempt **any Two** of the following:	**10**
	i)	Express $x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre Polynomial.	**5**
	ii)	Prove that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$ where $J_n(x)$ represents Bessel's Function of 1st Kind.	**5**
	iii)	Calculate the weight function for which the Eigen functions corresponding to distinct eigenvalues of the differential Equation $$y'' - 3y' = \lambda y, 0 < x < \pi, y(0) = y(\pi) = 0$$ are orthogonal.	**5**

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